

January 29, 1891.

Sir WILLIAM THOMSON, D.C.L., LL.D., President, in the Chair.

The Presents received were laid on the table, and thanks ordered for them.

The Bakerian Lecture was delivered as follows:—

BAKERIAN LECTURE.—“On Tidal Prediction.” By G. H. DARWIN, F.R.S., Plumian Professor and Fellow of Trinity College, Cambridge. Received December 16, 1890.

(Abstract.)

At most places in the North Atlantic the prediction of high and low water is fairly easy, because there is hardly any diurnal tide. This abnormality makes it sufficient to have a table of the mean fortnightly inequality in the height and interval after lunar transit, supplemented by tables of corrections for the declinations and parallaxes of the disturbing bodies. But when there is a large diurnal inequality, as is commonly the case in other seas, the heights and intervals, after the upper and lower lunar transits, are widely different; the two halves of each lunation differ much in their characters, and the season of the year has great influence. Thus simple tables, such as are applicable in the absence of diurnal tide, are of no avail.

The tidal information supplied by the Admiralty for such places consists of rough means of the rise and interval at spring and neap, modified by the important warning that the tide is affected by diurnal inequality. Information of this kind affords scarcely any indication of the time and height of high and low water on any given day, and must, I should think, be almost useless.

This is the present state of affairs at many ports of some importance, but at others a specially constructed tide-table for each day of each year is published in advance. A special tide-table is clearly the best sort of information for the sailor, but the heavy expense of prediction and publication is rarely incurred except at ports of first-rate commercial importance.

There is not, to my knowledge, any arithmetical method in use of computing a special tide table which does not involve much work and expense. The admirable tide-predicting instrument of the Indian

Government renders the prediction comparatively cheap, yet the instrument can hardly be deemed available for the whole world, and the cost of publication is so considerable that the instrument cannot, or at least will not, be used for many ports at remote places. It is not impossible, too, that national pride may deter the naval authorities of other nations from sending to London for their predictions, although the instrument may, I believe, be used on the payment of certain fees.

The object, then, of the present paper is to show how a general tide table, applicable for all time, may be given in such a form that any one with an elementary knowledge of the Nautical Almanac may, in a few minutes, compute two or three tides for the days on which they are required. The tables are also such that a special tide-table for any year may be computed with comparatively little trouble.

Any tide-table necessarily depends on the tidal constants of the particular port for which it is designed, and it is supposed in the paper that the constants are given in the harmonic system, and are derived from the reduction of tidal observations. Where the observation has been by tide-gauge, the process of reduction is that explained in the Report to the British Association for 1883, but where the observations are only taken at high and low water, a different process becomes necessary. I have given in a previous paper a scheme of reduction in these cases.*

At ports not of first-rate commercial importance observation has rarely been by tide-gauge, and thus it is exactly at those ports, where the method of this paper may prove most useful, that we are deprived of the ordinary method of harmonic analysis. On this account I regard the previous paper as preliminary to the present one, although the two are logically independent of one another.

In the harmonic method the complete expression for the height of water at any time consists of a number of terms, each of which involves some or all of the mean longitudes of moon, sun, lunar and solar perigees; there are also certain corrections, depending on the longitude of the moon's node. The variability of the height of water depends principally on the mean longitudes of the moon and the sun and to a subordinate degree on the longitude of lunar perigee and node, for the solar perigee is sensibly fixed. There are, therefore, two principal variables, and two subordinate ones. This statement suggests the construction of a table of double entry for the variability of tide due to the principal variables, and of correctional tables for the subordinate ones; and this is the plan developed in the paper.

The mean longitudes of the moon and sun are not, however, convenient as variables, and accordingly the principal variables in the

* 'Roy. Soc. Proc.,' 1890, vol. 48, p. 278.

tables are the time of moon's transit and the time of year; whilst the subordinate variables are the moon's parallax and the longitude of her node.

The tide-table, then, consists of the interval after moon's transit and height of high and low water, together with nodal and parallactic corrections, computed for every 20^m of moon's transit, and for about every ten days in the year. Each table serves for the two times of year at which the sun's longitude differs by 180°, and they may be used without interpolation. The nodal correctional terms consist of two times and of two heights, which are to be multiplied by the cosine and sine of the longitude of the moon's node, to give the total nodal corrections to the interval and height. The parallactic correctional terms consist of a time and a height, which are to be multiplied by the excess above, or defect below 57' of the moon's parallax at moon's transit to give the total parallactic corrections to the interval and height.

I had hoped that less elaborate tables might have sufficed, but it appeared that, at a station with very large diurnal inequality, the changes during the lunation, and with the time of year, in the interval and height are so abrupt and so great, that short tables would give very inaccurate results, unless used with elaborate interpolations. It is out of the question to suppose that a ship's captain would or could carry out these interpolations, and it is therefore proposed to throw the whole of that work on to the computer of the table.

Such a paper as this can only be deemed complete when an example has been worked out to test the accuracy of the tidal prediction, and when rules for the arithmetical processes have been drawn up, forming a complete code of instructions to the computer.

The port of Aden was chosen for the example, because its tides are more complex and apparently irregular than those of any other place which, as far as I know, has been thoroughly treated.

The arithmetic of the example was long, and was re-arranged many times. An ordinary computer is said to work best when he is ignorant of the meaning of his work, but in this kind of tentative work a satisfactory arrangement cannot be attained without a full comprehension of the reason of the method. I was therefore fortunate in securing the enthusiastic assistance of Mr. J. W. F. Allnutt, and I owe him my warm thanks for the laborious computations he has carried out. After computing fully half the original table, he made a comparison for the whole of 1889 of our predictions with those of the Indian Government. Without going into the details of this comparison, it may be mentioned that the probable error of the discrepancy between the two tables was 9^m in time, and 1.2 inches in the height of high water; that there were reasons to expect some systematic difference between the two calculations, and that all the considerable

errors of time fall on those very small rises of water which are of frequent occurrence at Aden.

I have made two other comparisons, one with the Indian predictions of 1887, and the other with actuality of 1884. In the latter case, when a few very small tides were omitted, the probable error was 7^m in the time, and 1·4 inches in height. It is concluded from these comparisons that, with good values for the tidal constants, the tables lead to excellent predictions, even better than are required for nautical purposes.

It is probable that this method may be applied to ports of secondary importance, where there are not sufficient data for very accurate determination of the tidal constants. Suggestions are made for very large abridgment of the tables in such cases, accompanied, of course, by loss of accuracy.

The question of how far to go in each case must depend on a variety of circumstances. The most important consideration is, I fear, likely to be the amount of money which can be expended on computation and printing; and after this will come the trustworthiness of the tidal constants, and the degree of desirability of an accurate tide-table. The aim of the paper has been to give the tables in a simple form, and if, as seems certain, the mathematical capacity of an ordinary ship's captain will suffice for the use of the tables, whether in full or abridged, I have attained the principal object in view.

Presents, January 29, 1891.

Transactions.

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